

8.6.1 EXERCISES

For a link to all of the additional resources available for this section, click [OSttS Chapter 8 materials](#).

In Exercises 1 - 6, find only the *form* needed to begin the process of partial fraction decomposition. Do not create the system of linear equations or attempt to find the actual decomposition.

1. $\frac{7}{(x-3)(x+5)}$

2. $\frac{5x+4}{x(x-2)(2-x)}$

3. $\frac{m}{(7x-6)(x^2+9)}$

4. $\frac{ax^2+bx+c}{x^3(5x+9)(3x^2+7x+9)}$

5. $\frac{\text{A polynomial of degree } < 9}{(x+4)^5(x^2+1)^2}$

6. $\frac{\text{A polynomial of degree } < 7}{x(4x-1)^2(x^2+5)(9x^2+16)}$

In Exercises 7 - 18, find the partial fraction decomposition of the following rational expressions.

For help with these exercises, click one or more of the resources below:

- [Finding a partial fraction decomposition \(distinct linear factors\)](#)
- [Finding a partial fraction decomposition \(repeated linear factors\)](#)
- [Finding a partial fraction decomposition \(irreducible quadratic factor\)](#)
- [Finding a partial fraction decomposition \(long division required\)](#)

7. $\frac{2x}{x^2-1}$

8. $\frac{-7x+43}{3x^2+19x-14}$

9. $\frac{11x^2-5x-10}{5x^3-5x^2}$

10. $\frac{-2x^2+20x-68}{x^3+4x^2+4x+16}$

11. $\frac{-x^2+15}{4x^4+40x^2+36}$

12. $\frac{-21x^2+x-16}{3x^3+4x^2-3x+2}$

13. $\frac{5x^4-34x^3+70x^2-33x-19}{(x-3)^2}$

14. $\frac{x^6+5x^5+16x^4+80x^3-2x^2+6x-43}{x^3+5x^2+16x+80}$

15. $\frac{-7x^2-76x-208}{x^3+18x^2+108x+216}$

16. $\frac{-10x^4+x^3-19x^2+x-10}{x^5+2x^3+x}$

17. $\frac{4x^3-9x^2+12x+12}{x^4-4x^3+8x^2-16x+16}$

18. $\frac{2x^2+3x+14}{(x^2+2x+9)(x^2+x+5)}$

19. One of the common algebraic errors the authors find students make is something along the lines of

$$\frac{8}{x^2-9} \neq \frac{8}{x^2} - \frac{8}{9}$$

Think about why if the above were true, this section would have no need to exist.

8.6.2 ANSWERS

1. $\frac{A}{x-3} + \frac{B}{x+5}$
2. $\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$
3. $\frac{A}{7x-6} + \frac{Bx+C}{x^2+9}$
4. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{5x+9} + \frac{Ex+F}{3x^2+7x+9}$
5. $\frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3} + \frac{D}{(x+4)^4} + \frac{E}{(x+4)^5} + \frac{Fx+G}{x^2+1} + \frac{Hx+I}{(x^2+1)^2}$
6. $\frac{A}{x} + \frac{B}{4x-1} + \frac{C}{(4x-1)^2} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{9x^2+16}$
7. $\frac{2x}{x^2-1} = \frac{1}{x+1} + \frac{1}{x-1}$
8. $\frac{-7x+43}{3x^2+19x-14} = \frac{5}{3x-2} - \frac{4}{x+7}$
9. $\frac{11x^2-5x-10}{5x^3-5x^2} = \frac{3}{x} + \frac{2}{x^2} - \frac{4}{5(x-1)}$
10. $\frac{-2x^2+20x-68}{x^3+4x^2+4x+16} = -\frac{9}{x+4} + \frac{7x-8}{x^2+4}$
11. $\frac{-x^2+15}{4x^4+40x^2+36} = \frac{1}{2(x^2+1)} - \frac{3}{4(x^2+9)}$
12. $\frac{-21x^2+x-16}{3x^3+4x^2-3x+2} = -\frac{6}{x+2} - \frac{3x+5}{3x^2-2x+1}$
13. $\frac{5x^4-34x^3+70x^2-33x-19}{(x-3)^2} = 5x^2-4x+1 + \frac{9}{x-3} - \frac{1}{(x-3)^2}$
14. $\frac{x^6+5x^5+16x^4+80x^3-2x^2+6x-43}{x^3+5x^2+16x+80} = x^3 + \frac{x+1}{x^2+16} - \frac{3}{x+5}$
15. $\frac{-7x^2-76x-208}{x^3+18x^2+108x+216} = -\frac{7}{x+6} + \frac{8}{(x+6)^2} - \frac{4}{(x+6)^3}$
16. $\frac{-10x^4+x^3-19x^2+x-10}{x^5+2x^3+x} = -\frac{10}{x} + \frac{1}{x^2+1} + \frac{x}{(x^2+1)^2}$
17. $\frac{4x^3-9x^2+12x+12}{x^4-4x^3+8x^2-16x+16} = \frac{1}{x-2} + \frac{4}{(x-2)^2} + \frac{3x+1}{x^2+4}$
18. $\frac{2x^2+3x+14}{(x^2+2x+9)(x^2+x+5)} = \frac{1}{x^2+2x+9} + \frac{1}{x^2+x+5}$